# Algebra 2/Trig Apps: Chapter 5 Quadratics Packet

In this unit we will:

- Determine what the parameters a, h, and k do in the vertex form of a quadratic equation
  $y = a(x - h)^2 + k$
- Determine the properties (vertex, axis of symmetry, shape) of the quadratic from looking at the coefficients in standard form $y = ax^2 + bx + c$
- Determine the roots of a quadratic function by factoring.
- Determine the roots of a quadratic function by the quadratic formula.
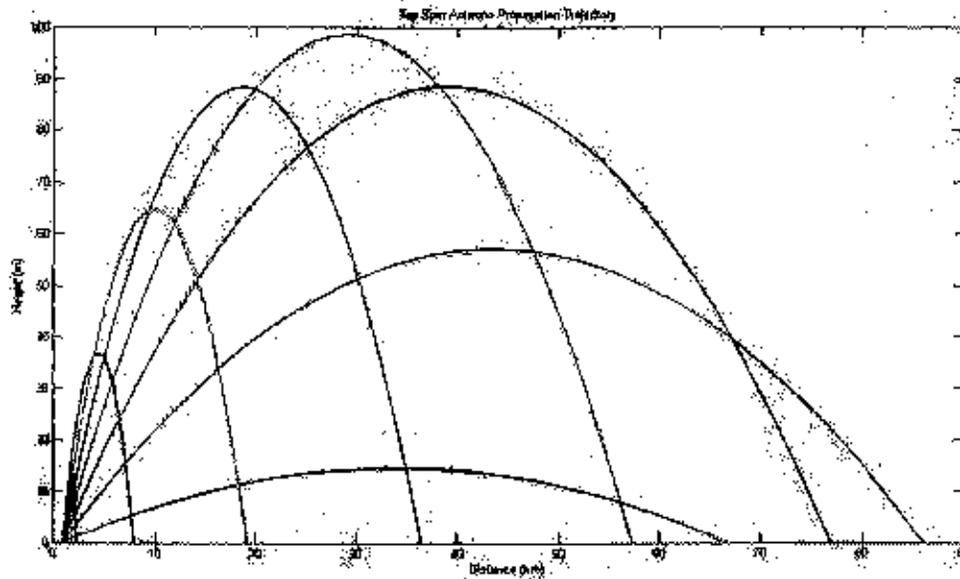
The reason why we call this form **"vertex form"** is because it indicates where the vertex is.

$$\text{Example: } y = -2(x-4)^2 + 3$$
$$y = a\ (x-h)^2\ + k$$

Vertex (h, K)

a=   h=   k=

The vertex is therefore (   ).

For each of the following, give the coordinates of the vertex.

1. $y = 4(x-3)^2 + 5$ Vertex:

2. $y = 2(x+3)^2 + 6$ Vertex:

3. $y = -3(x-2)^2 - 8$ Vertex:

4. $y = x^2 + 5$ Vertex:

For each of the following, write the equation (in vertex form) of the parabola given the vertex and the value of a.

5. Vertex : (2, 5) and a = 2   Equation:

6. Vertex : (-3, 4) and a = 1   Equation:

7. Vertex : (-8, 5) and a = -1   Equation:

8. Vertex : (6, 0) and a = -5   Equation:

A2T Apps: Properties of a Quadratic Function in Standard Form
SWBAT Convert Vertex Form Equations to Standard Form

Warm – Up
1. Answer each of the following questions.
2. Your answer is written in vertex form $y = a(x - h)^2 + k$
3. Compare your answers with your partner.
4. Be prepared to share your reasoning

a) Write the quadratic equation for the parabola with vertex (-5, 3) and "a" = 3.
b) Determine whether the parabola opens upward or downward
c) Is the parabola wider or skinnier compared to a parabola with "a" = $\frac{1}{3}$?

I. Standard Form of a Quadratic Equation: $f(x) = ax^2 + bx + c$
- a is always the coefficient of the x^2 (quadratic) term.
- b is always the coefficient of the x (linear) term.
- c is always the coefficient of the "no x" (constant) term.

Determine a, b, and c of each of the following:

	Re-arrange to be in standard form	a	b	c
1. $y = -4x + x^2 - 5$				
2. $y = x^2 - 6x + 9$				
3. $y = x^2 - 4$				
4. $y = 2x^2 - 7x - 4$				
5. $y = x^2 - 5x + 6$				
6. $y = x^2 - 4x$				

II. Converting an Equation from Vertex Form to Standard Form

In order to convert an equation from vertex form $y = a(x - h)^2 + k$ to standard form $y = ax^2 + bx + c$, FOIL the squared term first, distribute the value of the coefficient afterwards if there is one, and then combine any like terms.

Example 1: $y = 3(x - 2)^2 + 5$ **Example 2:** $y = -2(x + 1)^2 - 4$

Practice:

Convert each of the following equations in vertex form to the equivalent equation in standard form $y = ax^2 + bx + c$ by multiplying and simplifying.

7. $y = (x + 4)^2 + 5$

8. $y = 4(x - 3)^2 - 1$

9. $y = 2(x + 3)^2 + 6$

10. $y = -3(x - 2)^2 - 8$

SUMMARY

To convert from Vertex Form, $y = a(x - h)^2 + k$ to Standard Form $y = ax^2 + bx + c$

$y = 2(x - 3)^2 - 4$
$\quad = 2[(x - 3)(x - 3)] - 4$ *Rewrite the term to be squared as the product of two binomials*
$\quad = 2(x^2 - 6x + 9) - 4$ *FOIL the squared term first*
$\quad = 2x^2 - 12x + 18 - 4$ *Distribute the value of the coefficient*
$\quad = 2x^2 - 12x + 14$ *Combine like terms*

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A2T Apps: Axis of Symmetry, Vertex, and Opening of a Quadratic Function
SWBAT – Determine the Axis of Symmetry, Vertex and Opening of a Parabola given equation in
   standard form.

Warm – Up

LAB ACTIVITY   EXPLORE THE AXIS OF SYMMETRY

Every graph of a quadratic function is a parabola that is symmetric
about a vertical line through its vertex called the *axis of symmetry*.

There is a relationship between a and b in the quadratic function and
the equation of the axis of symmetry.
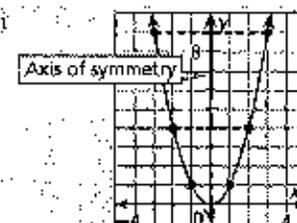
1. Complete the table.

Function	$y = 1x^2 - 2x - 3$	$y = -2x^2 - 8x - 6$	$y = -1x^2 + 4x$
Graph			
a	1	▦	■
b	–2	▦	■
$\dfrac{b}{a}$	▦	▦	■
Axis of Symmetry (from graph)	$x = 1$	▦	▦

2. Compare the axis of symmetry with $\frac{b}{a}$ in your chart. What can you multiply $\frac{b}{a}$ by to get the number in the
   equation of the axis of symmetry? (*Hint:* Write and solve an equation to find the value.)
   Check your answer for each function.

   _____

3. Use your answer from Problem 2 to complete the equation of the axis of symmetry of a quadratic

   function. $x =$ _____

   • If a>0 (is positive) then the vertex has a **minimum point** (is "happy.")
   • If a<0 (is negative) then the vertex has a **maximum point** (is "sad.")
   • **Axis of symmetry:** $x = -\frac{b}{2a}$.

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- The **Vertex** has same x-coordinate as the axis of symmetry. Find the y-coordinate obtained by substituting x into the original equation.

Practice: Determine the axis of symmetry and vertex of each of the following.

1. $y = x^2 + 2x + 1$ **2.** $y = x^2 + 4x + 10$ **3.** $y = x^2 - 6x + 5$

4. $y = x^2 - 10x - 4$ **5.** $y = 2x^2 + 4x - 3$ **6.** $y = 2x^2 - 10x + 1$

7. $y = -3x^2 + 8x - 5$ **8.** $y = -4x^2 + x + 1$ **9.** $y = x^2 + 2x + 5$

10. $y = x^2 + 3x$ **11.** $y = x^2 + 5$ **12.** $y = 2x - x^2$

Summary

Finding the Axis of Symmetry by Using the Formula

FORMULA	EXAMPLE
For a quadratic function $y = ax^2 + bx + c$, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$	$y = 2x^2 + 4x + 5$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(2)} = -1$ The axis of symmetry is $x = -1$.

Algebra2/Trig: Factoring a Quadratic Expression
SWBAT – Factor a Quadratic Expression

Warm – Up

Use the distributive property to find each product.
Multiply each polynomial

1. $2x(3x^5 - 2x + 1)$
2. $4xy\,(3x + 6y - 7)$

There are a number of different ways to factor a quadratic expression.

Greatest Common Factor (GCF)
The Greatest Common Factor (GCF) is the largest numeric value and variable power that can be divided out of a polynomial. **Always start by factoring out any GCF!**

Example 1: Factor $8x^4 - 12x^3 - 16x^2$

You can treat the coefficients separately from each variable. First, look for the largest value that is a factor of 8, 12, and 16. 4 is the largest value.

Then, for each variable, find the greatest power of that variable that can be divided out of that variable. The greatest power of x that can be divided out of x^4, x^3, and x^2 is x^2.
Think of what will remain when you divide each term by the GCF. Writing the problem this way may help you see what the remaining factor is. Recall that when you divide powers of a variable, you subtract the exponents.

Write the GCF on the left, and the remaining factor in parentheses on the right.

Be Careful of THIS!!
Example 2: $16x^4 - 12x^3 - 4x^2$

Example 1: Factor each of these by determining the GCF.

1. $12ab + 30ac$
2. $p + prt$
3. $12x^2 y^3 - 18xy^4$
4. $6b^2 - 36b^3$

5. $x^3 - 3x^2 + x$
6. $2x^4 - 6x^2 + 12x^6$

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Example 2: Factor by Grouping

When a polynomial has four terms, make two groups and factor out the GCF from each group.

Factor $8x^3 + 6x^2 + 20x + 15$.

Step 1: Group terms that have common factors.

Step 2: Identify and factor the GCF out of each group.

Step 3: Factor out the common binomial factor.

Check:

Factor each polynomial filling in the blanks.

1. $(18x^3 + 15x^2) + (24x + 20)$

    GCF is ☐   GCF is ☐

    ☐$(6x + 5) + $☐$(8x + 5)$

    $($☐$)(6x + 5)$

2. $(10a^3 - 15a^2) + (12a - 18)$

    GCF is ☐   GCF is ☐

    ☐$(2a - 3) + $☐$(2a - 3)$
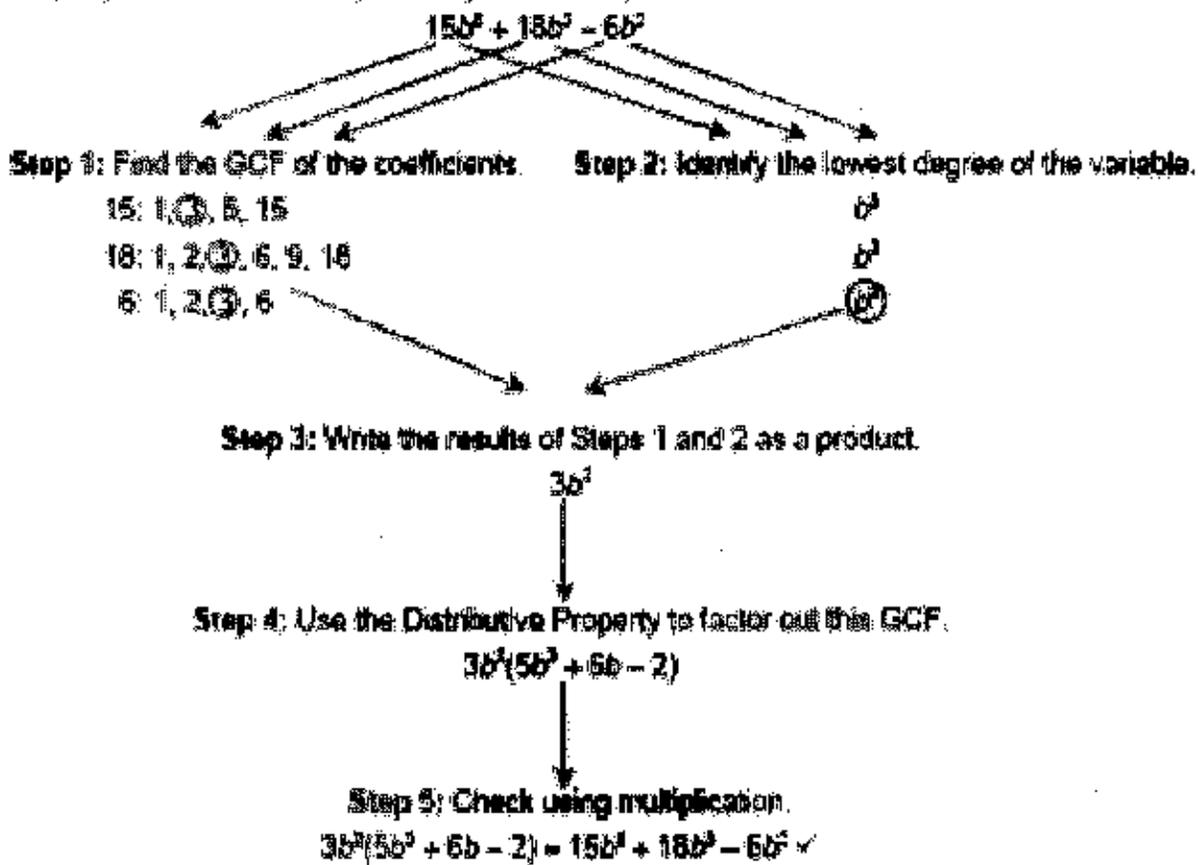
    $($☐$)(2a - 3)$

Factor each polynomial by grouping.

3) $21x^3 + 12x^2 + 14x + 8$

4) $40x^3 - 50x^2 + 12x - 15$

SUMMARY

To factor a polynomial by using the GCF, think of using the Distributive Property in reverse. More specifically, follow the procedure shown here.

$$15b^3 + 18b^2 - 6b^2$$

Step 1: Find the GCF of the coefficients.   Step 2: Identify the lowest degree of the variable.

15: 1, ③, 5, 15 b^3

18: 1, 2, ③, 6, 9, 18 b^2

6: 1, 2, ③, 6   ⓑ

Step 3: Write the results of Steps 1 and 2 as a product.

$$3b^2$$

Step 4: Use the Distributive Property to factor out this GCF.

$$3b^2(5b^2 + 6b - 2)$$

Step 5: Check using multiplication.

$$3b^2(5b^2 + 6b - 2) = 15b^4 + 18b^3 - 6b^2 \checkmark$$

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Algebra 2 Trig/ APPS
Practice/Homework

1) $12x^2 - 16$

2) $-15v^2 - 25$

3) $8n + 4$

4) $-2k^4 + 3k^6$

5) $-a + a^2$

6) $5x^4 - 2x^3$

7) $63x^4y^3 - 36x^2y^4 + 45xy^4$

8) $56v^5u^2 - 7v^5u - 56v^5$

9) $18x^2 + 10x^2y^2 + 12x^3$

10) $8v^5 + 2v^3u - 2v^2$

11) $50xy^4 - 100x^2y + 20xy$

12) $40a^7b - 50a^4b - 5a^4$

13) $9a^2b^4c + 9a^3b^3c$

14) $-21hjk + 7h^2$

Factor each polynomial by grouping. Check your answer.

15) $x^3 + 4x^2 + 2x + 8$

16) $2m^3 + 4m^2 + 6m + 12$

17) $6x^3 + 4x^2 + 3x + 2$

18) $7r^3 - 35r^2 + 6r - 30$

19) $4b^3 - 6b^2 + 10b - 15$

20) $6a^3 - 9a^2 - 12 + 8a$

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Algebra2/Trig Apps: Difference of Perfect Squares Factoring
SWBAT: Factor the Difference of Perfect Squares (DOPS)

Warm – Up
1. Factor Using the GCF $\quad -90b^5 + 30b^4 - 50b^3$

   A) $10b^4(-9b^2 + 3b - 5)$
   B) $10b^3(-9b^2 + 3b - 5)$
   C) $10b^3(-18b^2 + 6b - 10)$
   D) $10b^4(-9b^3 + 3b - 5)$

2. Multiply $(x + 3)(x - 3)$

Recognize a difference of two squares: the coefficients of variable terms are perfect squares, powers on variable terms are even, and there is a MINUS sign between them.

Example:
Factor $100x^6 - 9y^2$

This is a Difference of Perfect squares. 100 and 9 are perfect squares, and the powers of x and y are even. Determine what each term is a perfect square of. 100 is 10^2, 9 is 3^2, x^6 is $(x^3)^2$ and y^2 is the square of y.

Differences of perfect squares can be factored by the pattern $a^2 - b^2 = (a + b)(a - b)$. In this case, the a^2 is $100x^6$ so the a=$10x^3$ and b^2 is $9y^2$ so b=3y.

Careful! Don't forget that "1" is a perfect square!

Examples:

1. $x^2 - 9$

2. $a^2 - 4b^2$

3. $x^2 - 16$

4. $16 - x^2$

5. $4x^2 - y^2$

6. $4a^2 - 9$

7. $49a^4 - 81b^2$

8. $\dfrac{1}{9}m^2 - \dfrac{9}{16}n^4$

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Practice: Factor each of the following.

1) $25x^2 - 9$

2) $25r^2 - 4$

3) $16n^2 - 1$

4) $25b^2 - 16$

5) $v^2 - 16$

6) $x^2 - 9$

7) $a^2 - 4$

8) $4n^2 - 9$

9) $9m^2 - 1$

10) $4r^2 - 1$

11) $9x^2 - 25$

12) $9b^2 - 4$

Summary

Factor: $x^2 - 9$

Both x^2 and 9 are perfect squares. Since subtraction is occurring between these squares, this expression is the difference of two squares.

What times itself will give x^2 ? The answer is x.
What times itself will give 9 ? The answer is 3.

These answers could also be negative values, but positive values will make our work easier.

The factors are $(x + 3)$ and $(x - 3)$.
Answer: $(x + 3)(x - 3)$ or $(x - 3)(x + 3)$ (order is not important)

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Algebra2/Trig Apps: Factoring Quadratic Trinomials of the form $x^2 + bx + c$

SWBAT: Factor quadratic trinomials of the form $x^2 + bx + c$.

Warm – Up

1. Factor by Grouping. $24r^3 + 6r^2 - 28r - 7$

   A) $(6r^2 - 7)(4r - 1)$
   B) $(6r^2 - 7)(6r^2 + 1)$
   C) $(6r^2 - 7)(4r + 1)$
   D) $(6r^2 + 7)(4r - 7)$

2. Factor using GCF. $-90b^5 + 30b^4 - 50b^3$
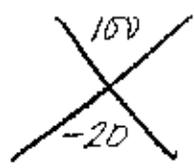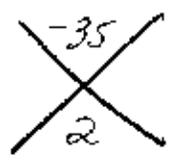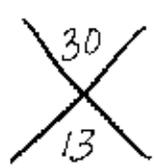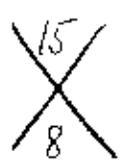
   A) $10b^4(-9b^2 + 3b - 5)$
   B) $10b^3(-9b^2 + 3b - 5)$
   C) $10b^3(-18b^2 + 6b - 10)$
   D) $10b^4(-9b^3 + 3b - 5)$

Mini-Lesson

Do you recognize the pattern???
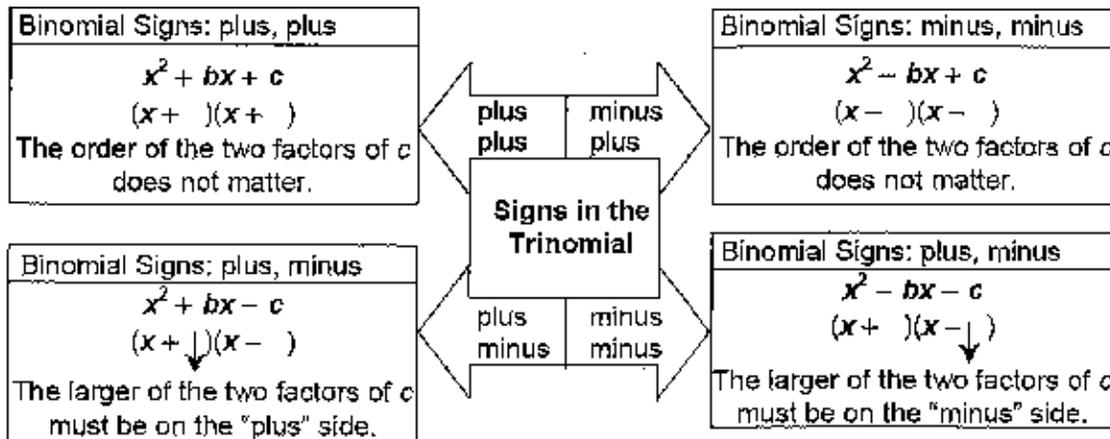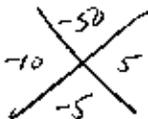

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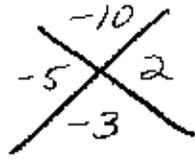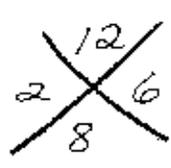
You Try!!! Complete the "Diamond"

Multiply

$(x + 2)(x + 5) =$ _____ = _____

✓ Notice the constant term in the trinomial; it is the product of the constants in the binomials.
✓ You can use this fact to factor a trinomial into its binomial factors.
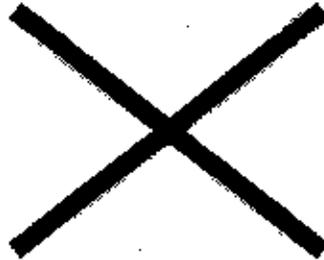   (Find two factors of c that add up to b)

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<u>Summary:</u>

Binomial Signs: plus, plus
$x^2 + bx + c$
$(x+\quad)(x+\quad)$
The order of the two factors of c does not matter.

Binomial Signs: minus, minus
$x^2 - bx + c$
$(x-\quad)(x-\quad)$
The order of the two factors of c does not matter.

Binomial Signs: plus, minus
$x^2 + bx - c$
$(x+\downarrow)(x-\quad)$
The larger of the two factors of c must be on the "plus" side.

Binomial Signs: plus, minus
$x^2 - bx - c$
$(x+\quad)(x-\downarrow)$
The larger of the two factors of c must be on the "minus" side.

Signs in the Trinomial

plus plus	minus plus
plus minus	minus minus

Example: Factor: $x^2 - 5x - 50$

$(x-10)(x+5)$

$ax^2 + bx + c$

Example 1: First Sign is Positive and Last Sign is Positive

Factor: $x^2 + 6x + 8$ Factor: $x^2 + 5x$

Answer: (   ) (   )   Answer: (   ) (   )

Practice 1: Factor.

1. $x^2 + 5x + 6$ **2.** $x^2 + 8x + 12$ **3.** $x^2 + 6x + 5$

Answer: (   ) (   )   Answer: (   ) (   )   Answer: (   ) (   )

4. $x^2 + 6x + 9$ **5.** $x^2 + 10x + 21$ **6.** $x^2 + 11x$

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Example 2: First Sign is Negative and Last Sign is Positive

Factor: $x^2 - 10x + 24$ Factor: $x^2 - 7x$

Answer: (   ) (   )   Answer: (   ) (   )

Practice 2: Factor.

7. $x^2 - 8x + 15$ 8. $x^2 - 6x + 8$ 9. $x^2 - 7x + 10$

Answer: (   ) (   )   Answer: (   ) (   )   Answer: (   ) (   )

10. $x^2 - 5x + 6$ 11. $x^2 - 13x + 40$ 12. $x^2 - 6x$

Example 3: First Sign is Positive or Negative and Last Sign is Negative
Factor: $x^2 + x - 20$

Answer: (   ) (   )

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Practice 3: Factor

13. $x^2 + 2x - 15$

Answer: (   ) (   )

14. $x^2 + 3x - 10$

Answer: (   ) (   )

15. $x^2 + 6x - 40$

Answer: (   ) (   )

16. $x^2 - 2x - 3$

17. $x^2 - 2x - 15$

18. $x^2 - 2x - 48$

Challenge Problem:

1) If $x + 2$ is a factor of $x^2 + bx + 10$, what is the value of b?

2) Factor: $x^4 + 18x^2 + 81$

ALGEBRA 2 TRIG/APPS Homework

1) $k^2 + 6k - 27$

2) $x^2 - 4x - 21$

3) $k^2 - 4k - 32$

4) $b^2 + 3b - 40$

5) $v^2 - 10v + 9$

6) $m^2 + 15m + 54$

7) $a^2 + a - 42$

8) $6r^2 - 66r + 60$

9) $r^2 + 14r + 45$

10) $a^2 + 8a + 7$

11) $x^2 + 5x + 6$

12) $x^2 + 5x + 4$

13) $v^2 + 10v + 16$

14) $x^2 - 5x - 24$

15) $p^2 + 3p - 28$

16) $a^2 - 5a - 6$

17) $v^2 + 11v + 30$

18) $n^2 + 2n - 8$

19) $x^2 - 7x + 6$

20) $b^2 - 15b + 54$

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ALGEBRA 2 TRIG/APPS

"REVIEW FOR TEST"
SWBAT: Apply their knowledge on Factoring

Station # 1
Common Monomial Factors (GCF)

Factor.

1) $9x^2 - 21x^5$ **2)** $4x^3 - 6x^2 + 10x$ **3)** $-12y^4z + 28y^5z + 40y^4zx$

Factor by Grouping.

4) $5x^3 - 15x^2 + 4x - 12$ **5)** $9n^3 + 15n^2 + 12n + 20$

Station # 2
Difference of Two Squares "D.O.T.S"

Factor.

1) $x^2 - 49$ **2)** $36x^2 - y^2$ **3)** $64 - y^2$

4) $9a^2 - 121y^2$ **5)** $a^6 - 9b^{12}$ **6)** $25x^4 - 144y^2$

Station # 3
Factoring Trinomials "Diamond"

1) $x^2 + 21x + 20$

2) $x^2 - 10x + 24$

3) $x^2 + 3x - 18$

4) $x^2 - 7x + 12$

5) $x^2 - 6x - 27$

6) $x^2 - x - 56$

Station # 4
Factoring Completely

1) ~~ax² - a~~

2) $4a^2 - 36$

3) ~~12x² - ...y²~~

4) ~~9a - 36b~~

5) ~~8x² + 15x - 42~~

6) ~~x⁴ - ...x² - 40x²~~

Station # 5
<u>Word Problems</u>

1) The area of rectangle is represented by $x^2 + 9x + 18$. Find the binomials that could represent the lengths and width of the rectangle.

2) The Volume of rectangular prism is represented by $p^3 - 12p^2 + 35p$. Find the factors that would represent the length, width, and height of the rectangular prism.

Algebra2Trig Summary: Factoring Completely

Look for GCF
Factor it out!
(Don't throw it away!)

Difference of perfect squares?
$x^2 - a^2$

Difference of perfect cubes?
$x^3 \pm a^3$

Quadratic Trinomial

Quadratic trinomial
$ax^2 + bx + c$

4 term polynomial (or more)

$(x-a)(x+a)$

$(x \pm a)(x^2 \mp ax + a^2)$

$(x \quad)(x \quad)$
Look for two numbers that multiply to c and add to b

Splitting the Middle METHOD

FACTOR BY GROUPING
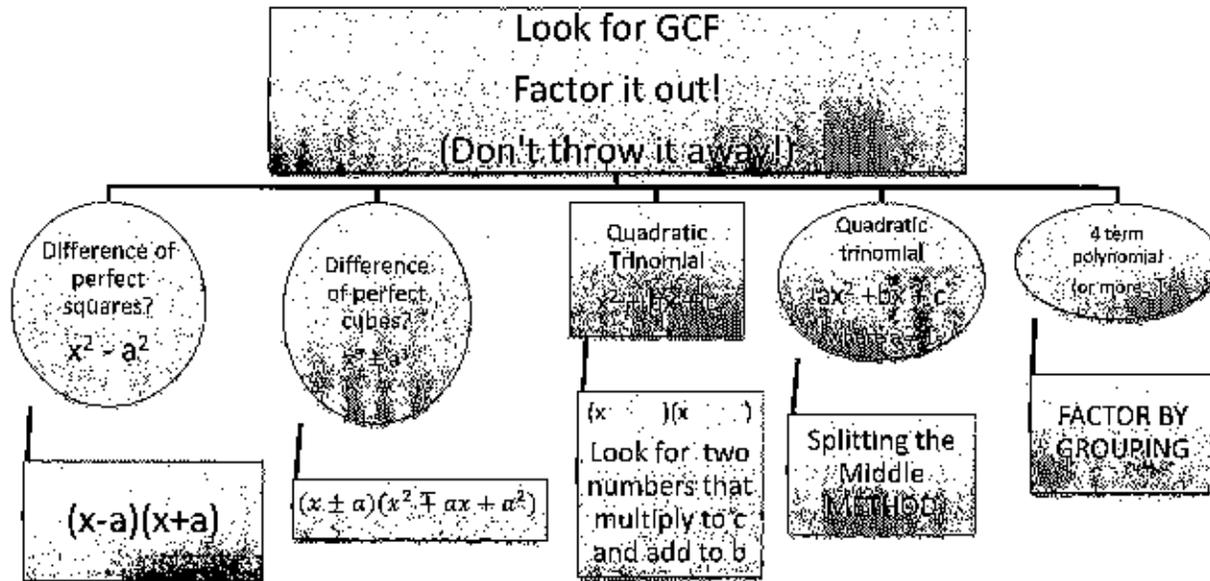
Everything, but in Vertex Form
$y = a(x - h)^2 + k$ **Vertex Form of a Quadratic**

- Axis of symmetry: $x = h$
- vertex: (h, k)
- a has the same effect on the appearance of the graph as it does in standard form.

Algebra2/Trig: Solving Quadratic Equations - Finding the Roots by Factoring

SWBAT: Solve Quadratic Equations by Factoring

WARM – UP

Factor each completely.

1) $r^2 + 16r + 63$

2) $m^2 - 6m - 27$

3) $p^2 - 13p + 40$

4) $x^2 + 7x - 18$

The **roots** or **zeros** of a quadratic equation are where the parabola
_____ the _____ axis. A _____ of a function is an x-
value that makes the function ( ) equal zero. A _____ of a function is also
the same as the x-intercept. The roots or zeros of a quadratic equation may
have two roots, one root or no roots.

$y = x^2 - x - 2$

$y = -2x^2 + 4x - 2$

$y = \frac{1}{4}x^2 + 1$

_____   _____   _____

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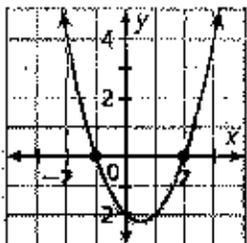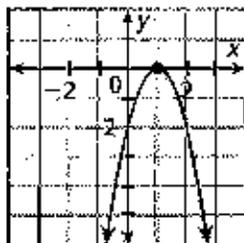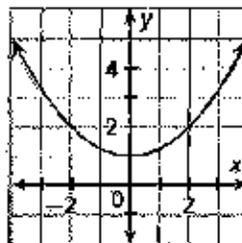
Solving (Roots) by Factoring

The **roots** (also called **zeros** or **solutions**) of a quadratic equation are where the graph of the equation hits the x-axis, or where y=0

In order to determine the roots of a quadratic, set the quadratic to 0, factor, and solve.

Example: Determine the roots of $y = x^2 + 8x - 20$

1. Set y = 0, or set the quadratic =0 if there is no y. $0 = x^2 + 8x - 20$
2. Factor the quadratic
3. Make a "t" chart.   Set each factor = 0
4. Solve each linear factor.

Practice:
Solve each of the following by factoring.

1) $x^2 + 11x + 28 = 0$

2) $p^2 + 7p + 10 = 0$

3) $n^2 - 11n + 30 = 0$

4) $m^2 + 12m + 35 = 0$

5) $v^2 + 7v = -12$

6) $a^2 = -49 - 14a$

7) $r^2 = 6r$

8) $b^2 + 2b = 35$

9) $n^2 - 4n = 12$

10) $x^2 + 16 = -10x$

11) $b^2 = b + 2$

12) $x^2 + 16 = -10x$

13) $x^2 - 4 = 0$

14) $k^2 + 9k = -14$

15) $x^2 - 13x = -7x$
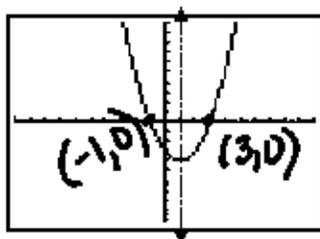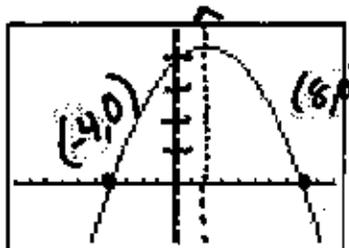
16) $n^2 + 7n = 0$

17) $p^2 + 5p = 14$

18) $r^2 = -2r + 8$

Name: _____ Date: _____

Summary

Exercise #1: The graph of the quadratic function $y = ax^2 + bx + c$ is given. For each graph identify the vertex, roots, Axis of symmetry, and determine if the vertex has a maximum or a minimum value.

(-1,0)   (3,0)

a) Turning point/Vertex __(1,-4)__

b) Roots __{-1, 3}__

c) Axis of symmetry __x = 1__

d) Max or min __min__

(-4,0)   (8,0)

a) Turning point/Vertex __(2,9)__

b) Roots __{-4, 8}__

c) Axis of symmetry __x = 2__
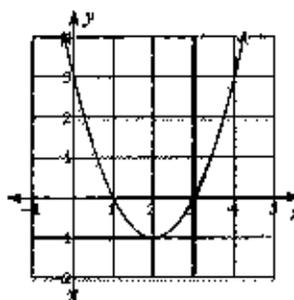
d) Max or min __max__

Exit Ticket

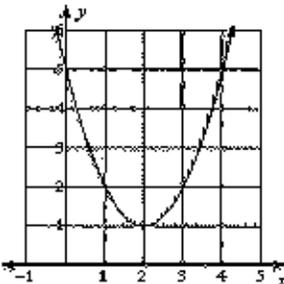Which graph represents the function: $y = x^2 - 4x + 5$

A)

C)

B)

D)

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